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TONTINES
A PRACTITIONER'S GUIDE TO MORTALITY-POOLED INVESTMENTS

RICHARD K. FULLMER, CFA

CFA Institute Research Foundation

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TONTINES

A Practitioner's Guide to Mortality-Pooled Investments

Richard K. Fullmer, CFA

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TONTINES: A PRACTITIONER'S GUIDE TO MORTALITY-POOLED INVESTMENTS

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PREFACE

My introduction to tontine thinking came in 2007 in the form of Ralph Goldsticker's *Financial Analysts Journal* article "A Mutual Fund to Yield Annuity-Like Benefits." I had previously heard of the tontine concept but never really thought about it in depth. The article piqued my intellectual curiosity as the idea—and all its potential variations—began to sink in.

Tontines became a hobby over the next few years. Scholarly materials on the subject seemed surprisingly scarce at the time, and it occurred to me that although tontines were a centuries-old product, modern tontine thinking was still in its infancy—a new frontier that had so far been explored only at its edges.

To my good fortune, Moshe Milevsky emerged as a leading expert on the subject and was kind enough to chat with me about the subject whenever our paths crossed. Milevsky introduced me to Michael Sabin, with whom I have since enjoyed a close collaboration in the study of tontines. Sabin introduced me to Jonathan Forman, a professor of law and fellow tontine researcher. I have learned much from these pioneers of modern tontine thinking.

The idea for this brief came from a conversation with Larry Siegel, CFA Institute Research Foundation Gary P. Brinson Director of Research. While chatting about other topics, the conversation eventually turned to my work on fair tontine design, which, in turn, led to a conversation about the potential benefit of producing an introductory practitioner's guide on the subject. The timing seemed right for three reasons. First, interest in the potential of tontines and tontine-like solutions is picking up, spurred by the so-called global retirement crisis. Second, the products are widely misperceived and poorly understood. Third, practitioner curricula on the topic, whether investment or actuarial, are scarce if they exist at all. So, there was both a reason and a need.

Mortality-pooled investing, such as with tontines, requires a bit of a paradigm shift relative to traditional investing. Although this brief is far from a complete handbook on the subject, it aims to serve as a practitioner's basic introduction.

If most of what you know about tontines came from a fictional novel, a film, a newspaper article, or an episode of *The Simpsons*, rest assured that you are not the only one. But this does not have to be the case. Tontine research has come a long way since Ralph Goldsticker's insightful article initially launched me on my personal path of study. Yes, it is indeed possible to design a mutual fund to yield annuity-like benefits. But the limits of fair tontine design—with *fair* being the operative word here—extend far beyond. The fair tontine principle is more versatile than you might think. Perhaps one day tontine finance will become a specialty of its own. I hope so.

RKF

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It is impossible for me to imagine writing this text absent my research partnership with Michael Sabin. I thank him as well as Don Ezra, Manuel García-Huitron, and Arun Muralidhar for reviewing the text and providing insightful comments. I also thank Catherine Donnelly and Jonathan Forman for kindly exchanging thoughts and ideas about their research on the subject. A debt of gratitude is also owed to Moshe Milevsky, not only for teaching me much about pension finance through his writing but also for providing encouragement in the noble (and challenging!) pursuit of advancing “tontine thinking” beyond academia into the practitioner’s realm. Finally, I would like to thank Larry Siegel and the rest of the CFA Institute Research Foundation staff for their work and support in making this guide possible.

WHAT IS A TONTINE?

A tontine is a financial arrangement in which members form an asset pool and then, mutually and irrevocably, agree to receive payouts from it while living and—this is key—forfeit their accounts upon death, with the forfeiture proceeds apportioned among the surviving members. Payouts depend on investment performance and the mortality experience of the membership pool.

It is easy to see that all sorts of designs are possible. The simplest design may be a type of closed-end lottery pool, in which the tontine takes in contributions from some initial set of participants and pays out nothing until only one survivor remains, who then receives the entire pot (or perhaps until *N* survivors remain, who split the pot). Other designs blend elements of an investment and a lottery, offering periodic payouts such that survivors receive dividends that increase exponentially as other members die. However, tontines need not be lottery-like at all. Indeed, it is possible—and perhaps more broadly useful—to engineer them with annuity-like payouts, whether flat or growing. Furthermore, tontines need not be closed-end structures. The pool may be open-end and perpetual, continually taking in new members to replace those who have died. Other designs with tontine-like features go by such names as pooled annuity funds or group self-annuitization schemes.

The challenge is to make the arrangement fair. For example, consider a tontine in which the payout rates are the same regardless of a member's age or gender. Such a tontine would favor the young over the old and women over men since these demographic groups are more likely to survive longer and thus receive greater payouts. Unfair tontine designs are easy to dream up but are quite unpractical and arguably bad for society. This text focuses instead on *actuarially fair* tontine design—meaning a design that favors no person or group.[1]

WHY STUDY TONTINES?

Retirement is a hard problem to solve. If it were easy, there would be no chronic underfunding of pension plans. The financial status of government social security programs would be healthy rather than bleak. Individuals would feel confident in retiring while sustaining their desired lifestyles for the rest of their lives. Unfortunately, every solution to the problem has both benefits and drawbacks, and in each case, the drawbacks can be significant.

Tontines do not magically fix the problem, of course. But they do offer a unique value proposition—one that lies in the middle ground between investment and insurance solutions, with elements of both. They might be appealing to the following groups:

- Employers that wish to offer defined benefit–like employee pension plans that can never become underfunded
- Defined contribution plan sponsors that wish to offer participants an option that provides the assurance of annuity-like lifetime income while avoiding the fiduciary liability and counterparty risk associated with selecting an insurance company as guarantor
- Investors who wish to increase their returns without increasing investment risk

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[1]Actuarial science involves the pricing of risk on the basis of probabilities of loss (death, in this case) that are typically constructed from observed, or otherwise projected, statistical distributions. The term "actuarially fair" in this context refers to fairness in the risk–reward trade-off given a set of such probabilities.

- Anyone seeking the assurance of lifetime income with greater transparency and at lower cost than with insurance guarantees
- Policymakers who wish to encourage retiree participation in lifetime income solutions
- Governments that wish to create (or recreate) a market for lifetime income products in countries where annuity markets are nonexistent or dysfunctional

Naturally, there are important trade-offs to consider for each of these benefits. But the point is that tontines represent a new solution for addressing the global retirement challenge. They represent a new choice for employers, savers, and retirees, and choice is good.

LONGEVITY-RISK POOLING

There are two key distinctions between a tontine and an ordinary investment. First, tontine investments are generally irrevocable, because if they were not, members who discover they have a serious illness would have an incentive to defeat the risk-sharing purpose of the arrangement by withdrawing their account balances just before they died. Second, account balances are not transferred to a member's beneficiaries upon death. Instead, they are fairly apportioned among the surviving participants. Thus, monies forfeited by those who die increase the returns to those who survive. These extra returns are often referred to as "mortality credits." In this way, tontines allow members to collect lifetime income by collectively *self-pooling* longevity risk among themselves.

Risk pooling is powerful because although the lifespan of any individual member is highly uncertain, the lifespan of the group is much less uncertain. Tontines allow members to diversify away substantially all idiosyncratic longevity risk—the uncertainty associated with how long they will live compared with others in their demographic cohort and how much they can withdraw or spend without outliving their savings. The degree of diversification achieved depends largely on the size of the membership pool. The underlying principle is the law of large numbers: Diversification increases with the size of the membership pool.

Members still bear systematic mortality risk—the risk that the entire membership group lives longer or shorter than expected. However, this risk is mitigated in that adjustments to tontine payouts are made gradually over time. Should the membership die slower (faster) than expected, payouts adjust downward (upward). These continual adjustments, along with similar adjustments for investment performance, are the mechanism that keeps the tontine fully funded at all times and allows it to offer the assurance of lifetime payouts.[2] To anyone concerned about the sad state of underfunded pensions, these words should come as music to the ears!

With a tontine, then, members give up the ability to withdraw from the scheme at will but gain mortality credits on top of the scheme's underlying investment returns. As a result, tontine accounts can pay out to survivors more than an ordinary investment account can. Furthermore, tontines can provide annuity-like lifetime payouts at lower cost than comparable annuities by eliminating the costs of risk transfer. This combination makes them an attractive alternative for retirement income.

[2]The funding status of a tontine is a rather obscure notion. Tontines do not have funding liabilities. They are asset pools that pay out only what they can and no more. Thus, the present value of the pool's assets will (must!) always equal the present value of its future payouts, by definition. In this way, tontines are considered fully funded in an economic sense. *Fully funded* does not imply *adequately funded*. Adequacy with respect to any given payout objective requires a sufficient combination of contributions into the system and investment returns on those contributions.

A (VERY) BRIEF HISTORY OF TONTINES

Tontines were named after Lorenzo de Tonti, an Italian banker who proposed the arrangement to the French in 1653. For centuries, it was believed that Tonti invented the idea. However, it was recently discovered that Nicolas Bourey, a Belgian, proposed a similar scheme in Portugal 12 years earlier, in 1641.[3]

The history of tontines is as long as it is fascinating. To those interested in this history, McKeever (2009); Milevsky (2015); and Hellwege (2018) are useful references. For the concise purpose of this text, however, I shall boil the history down in the following way: Tontines were originally used by national governments (kings) to raise money in financing wars against other nations. In the earliest tontines, shareholders could select a nominee different from themselves as the person on whose life the contract was contingent. By the 19th century, private insurance companies determined that they, too, could issue tontines. By the end of that century, privately issued products had become quite popular in Europe and the United States. Sadly, success did not last long, because the products fell victim to misappropriation and fraud on the part of tontine issuers. As a result, regulators in several countries moved to ban the products early in the 20th century, and tontines largely faded away.

But that was then, and this is now. The tontine concept is enjoying renewed interest given its potential to alleviate the so-called global retirement crisis amid a recognition that the problems that led to the banning of tontines more than a century ago are perhaps no longer an issue. After all, investor protections have come a long way in the past century. Record keeping and auditing have improved. Financial assets are now held by independent custodians. Required disclosures are stricter.

[3]From McKeever in Hellwege (2018, p. 193).

ARE TONTINES LEGAL?

Tontines never did fade away completely. Most notably, they persisted in France as a niche product. The pension system in Sweden is explicitly tontine-like. Moreover, a slow trend toward acceptance of such longevity pooling arrangements has arisen over the past few decades. European Union member states now allow tontine offerings under the Second European Life Insurance Directive of 1990.[4] A legal tontine is now operating in South Africa. Australia appears to be moving in this direction with tontine-like group self-annuitization schemes. Canada announced a proposal to allow tontines in retirement accounts as part of its Budget 2019 reforms.

The legality in other jurisdictions is debatable. The conventional thinking among investment practitioners is that tontines are illegal in the United States. However, several tontine-like schemes have been operating in the United States for years. Some public pension plans (in Wisconsin, for example) operate with tontine-like characteristics. In the private sector, the CREF variable annuity—which appears to operate like a tontine in virtually every way except for its name—was first offered in 1952 and remains open for purchase even today. One might ask how this could be if tontines are supposedly not legal. But the question of whether these products are legal involves more than a yes or no answer.

McKeever (2009); Forman and Sabin (2015); and Hellwege (2018) are recommended reading

[4]From Hellwege (2018): Council Directive 90/619/EEC of 8 November 1990 on the coordination of laws, regulations, and administrative provisions relating to direct life assurance (29 November 1990, pp. 50–61).

for anyone interested in the legal history and regulatory status of tontines. To be sure, regulatory hurdles exist for any firm that seeks to bring these products to market. Yet the trend suggests that these hurdles may perhaps be overcome with the right design and, as Milevsky has pointed out, a very good lawyer.[5]

LITERATURE

The literature on tontines and tontine-like arrangements, such as pooled annuity funds and group self-annuitization schemes, is growing. A rough categorization follows:

- Historical references: See McKeever (2009); Milevsky (2015); and Hellwege (2018).
- Legal and regulatory: See McKeever (2009); Hellwege (2018); and Forman and Sabin (2015).
- Tontine design: See Goldsticker (2007); Sabin (2010, 2011); Donnelly, Guillén, and Nielsen (2013, 2014); Newfield (2014); Forman and Sabin (2015, 2016); Milevsky and Salisbury (2015, 2016); Sabin and Forman (2016); Weinert and Gründl (2016); Fullmer and Sabin (2018, 2019); and Bernhardt and Donnelly (2019).
- Other designs that include tontine-like characteristics: See Piggott, Valdez, and Detzel (2005); Stamos (2008); van de Ven and Weale (2008); Richter and Weber (2011); Denuit, Haberman, and Renshaw (2011); Qiao and Sherris (2013); Donnelly (2015); and Chen, Hieber, and Klein (2019).
- Price/cost comparison: See Milevsky, Salisbury, Gonzalez, and Jankowski (2018).
- Fair surrender value: See Weinert (2017).

[5]Hellwege (2018, p. 316).

THE FAIR TONTINE PRINCIPLE

A fundamental principle of tontines is that their design should be fair to all investors. This principle requires that forfeited balances be transferred to survivors in a manner such that no investor (and, therefore, no class of investor) is unfairly disadvantaged. But how? To help answer this question, let us start with the example of fair games.

Fair Games

A game or bet is fair to all parties if each party's expected gain is zero. Consider the classic coin flip in which a player wins if the coin lands on heads and loses if it lands on tails. This game is fair when the amount gained by winning is the same as the amount forfeited by losing. There is a 50% chance of each outcome, and thus the expected gain per dollar staked is $(\frac{1}{2} \times -1) + (\frac{1}{2} \times 1) = 0$. If the expected gain were greater than zero, the player would have an advantage. Similarly, if the expected gain were less than zero, the player would have a disadvantage.

Now say that the game involves rolling a 6-sided die in which the player wins with a roll of 6 and loses otherwise. Here, there is a $\frac{5}{6}$ chance of losing and a $\frac{1}{6}$ chance of winning. This game is fair when the amount gained by winning is 5 times more than the amount forfeited by losing because the expected gain per dollar staked in that case is $(\frac{5}{6} \times -1) + (\frac{1}{6} \times 5) = 0$.

We can generalize a fair game payoff as follows. Assume that a player puts up a stake of s in a game in which the probability of losing is q. For the game to be fair, the expected value from winnings must equal the expected value from losses. For each player, the following equation must hold each time the game is played:

$$-qs + (1 - q)c = 0.$$

The quantity $-qs$ represents the probabilistic expected loss from playing the game, and the quantity $(1 - q)c$ represents the probabilistic expected gain from playing the game, where c represents the payoff if the game is won. Solving for c, we get

$$c=\underbrace{\frac{qs}{1-q}}_{\text{Gain}}=\underbrace{\left(\frac{q}{1-q}\right)}_{\text{Gain rate}}s=rs.$$

The quantity c represents the amount gained by winning, and the quantity $r = q/(1 - q)$ represents the gain rate. The gain rate is 100% in the coin toss game because $0.5/(1 - 0.5) = 1$, or 100%. The gain rate is 500% in the die-rolling game because $\frac{5}{6} / (1 - \frac{5}{6}) = 5$, or 500%.

An interesting observation is that the expected value of playing the game is the same as the expected value of *not* playing the game. As a result, and assuming that we are not trying to appeal to anyone's gambling interests, there seems to be no economic incentive to play. But might circumstances exist in which playing is indeed economically beneficial? Hold this thought. I shall return to it later.

Fair Tontines

In a tontine, investors forfeit their balances when they die. Forfeited balances (losses) are apportioned and given to the surviving members in the form of mortality credits (gains). The objective is to ensure that the gains received by each and every survivor fairly offsets the amounts that they risk losing when they die.

The intuition for how this must be accomplished is remarkably similar to the simple games discussed earlier. By defining a member's current account balance as s, the probability that the member dies in a given period (also known as the mortality rate) as q, and the required mortality credit to be received upon surviving the period as c, we again arrive at

$$-qs + (1 - q)c = 0.$$

Now, the quantity $-qs$ represents the probabilistic expected value of forfeiture due to the risk of dying during the period and the quantity $(1 - q)c$ represents the probabilistic expected mortality credit if the member survives the period. Solving for c, we get

$$c=\frac{qs}{1-q}=\left(\frac{q}{1-q}\right)s=rs,$$

where $r = q/(1 - q)$ represents the *nominal tontine yield* and rs represents the *nominal tontine gain*, meaning the nominal yield, r, times the current account balance, s. Because q is bounded by the range $0 < q < 1$, r is a positive value. Because mortality rates increase with age, each member's nominal tontine yield also increases with age.

To be clear, tontine gains represent amounts received as a result of mortality pooling, over and above any investment returns that may also be earned. Thus, a tontine investment delivers both an investment yield and a tontine yield (i.e., mortality-credit yield).

Persistence

A member's account balance, s, will vary over time as a function of investment returns, tontine gains, and any payouts. Her mortality rate, q, will also vary over time as a function of the aging process and mortality improvements that may be enjoyed by her age cohort in the general population. To accommodate the desire that the tontine remain fair to all members throughout the life of the tontine pool, we can add a member subscript j and a periodic time subscript t to the formulas to arrive at the following fairness constraint:

$$-q_{j,t}s_{j,t}+(1-q_{j,t})c_{j,t}=0$$

for all j and for all t. Periods could be of any length. In practice, monthly periods are likely to be popular in order to accommodate more easily the monthly payouts that retirees usually desire.

Observations

Three observations stand out.

First, note that, similar to the fair games discussed earlier in which the expected value of playing is the same as the expected value of not playing, the expected cumulative return of an investment in a fair tontine account is the same as the expected cumulative return of the same investment in a regular account.[6] This principle is what enforces fairness; no one is advantaged or disadvantaged by entering the tontine.

Second, the nominal tontine gain, rs, represents an amount that a member probabilistically expects to receive, conditional on surviving. The actual tontine gain received will be a random amount that depends on which members die. When the number of members in the pool is large, however, the actual gain is likely to be close to the nominal gain that is expected. This is a result of the law of large numbers effect in diversifying idiosyncratic longevity risk.

Third, a surviving member's expected tontine gain depends only on his own balance, s, and his own probability of dying, q. This follows from the observation that his expected tontine gain is $rs = sq/(1 - q)$. It does not matter who else is participating: young or old, male or female, individual or couple, rich or poor. It also does not matter *how* others are participating: the size of their accounts, the investments they select, their trading activity, or the payout options they elect. The only things that matter are a member's own balance and own probability of death. This may seem surprising and perhaps even paradoxical, but it is true.[7]

FAIR VS. EQUITABLE

Can tontines be truly fair or merely equitable?

I have defined a "fair" tontine as one in which the expected value of the tontine gain or loss that a member experiences as a result of mortality pooling is zero for each member. That is, each member receives a fair bet in the probabilistic sense.

It has correctly been argued, however, that because a tontine pool with a finite life will unavoidably have a little money left over when the last member dies, the expected cumulative value of a member's tontine gains (while living) and losses (upon death) will be slightly negative, rather than zero. In the strictest sense, such a tontine would not be actuarially fair. However, it could still be equitable in that no member is disadvantaged.[8]

The situation is different in an open-end tontine that accepts new members in perpetuity. It has

[6]This statement ignores the fact that a tontine would likely charge a somewhat higher fee owing to additional administration costs, such as keeping track of who has died and processing forfeiture redistributions. Recall also that in a regular account, the account owner's beneficiaries inherit the account balance upon the owner's death, whereas the balance is forfeited in a tontine account.

[7]A number of authors have demonstrated this—notably, Sabin (2010), Donnelly et al. (2014), Milevsky and Salisbury (2016), and Fullmer and Sabin (2018).

[8]This argument is discussed in Milevsky and Salisbury (2016).

been argued that a strictly fair tontine is possible in that case because the pool never runs out of members and, therefore, no money is ever left over.[9]

I elect to use the term *fair* in this text to highlight the fundamental importance of the fairness concept in such products as these. Indeed, behavioral economists highlight the importance of the perception of fairness in lifetime income product adoption. For example, Shu, Zeithammer, and Payne (2018) found that a prospective buyer's perception of product fairness is the strongest predictor of a lifetime annuity purchase.

MORTALITY RATES

Mortality rates are fundamental to tontine design. Naturally, the rates to use should align with the subpopulation of the targeted membership. Age, gender, and country of residency are all relevant factors.

It makes sense to use a published mortality table provided by some independent, knowledgeable, and official source, such as the Society of Actuaries (or its counterpart in other countries) or a government entity. Such a table would likely incorporate some degree of adverse selection, because those who perceive themselves to be in worse-than-average health would be less likely to become members.[10] Furthermore, the table should also incorporate anticipated mortality improvements because mortality rates are not static but, rather, evolving (generally improving over the past several decades as life expectancies have risen).

Annuitant mortality tables (whether individual or group, depending on the product and target market) may be well suited to the task. Unlike annuities, however, a tontine's mortality table does not need to incorporate a longevity-risk reserve loading (that is, an additional factor to account for the risk that longevity will increase unexpectedly in the future). Because tontines guarantee nothing, no such risk reserve is necessary.

Table 1 illustrates using an excerpt of the 2012 Individual Annuitant Mortality (IAM) basic mortality table that has been projected to the year 2019 to account for mortality improvement.[11] This is a generational table, meaning that an individual's probability of death depends not only on age and gender but also on year of birth. It considers adverse selection but contains no reserve loading.[12] Because mortality improvements result in a decreasing probability of death (i.e., a longer life) as the birth year increases, the nominal tontine yield for individuals of a given age and gender decreases with each year that the tontine is in operation. In other words, the projected mortality rate table for the year 2020 will be different from the one shown in Table 1 for 2019. Although the rates shown here are annual, they may be transformed into another frequency, such as quarterly or monthly, using standard actuarial techniques, if desired. Note also that for accounts that are owned jointly by a couple, a mortality rate based on the last to die can be computed from this table using standard actuarial techniques.

[9]This argument is discussed in Fullmer and Sabin (2018).

[10]Adverse selection is more significant for tontines offered to individuals than it is for tontines offered as an automatic group benefit. This effect could be minimized through governmental legislation mandating that some portion of retirement savings be converted into a tontine or annuity with lifetime payouts.

[11]The projection is made using projection scale G2 as referenced in NAIC (2013).

[12]The 2012 Individual Annuity Reserving (IAR) mortality table is the reserve-loaded equivalent to the 2012 IAM table. Whereas annuity providers might, therefore, use the IAR table, tontine providers might instead use the IAM table. The IAR reserve loading is significant—10% at most ages.

TABLE 1. IAM RATE TABLE FOR 2019 UNDER PROJECTION SCALE G2 (EXCERPT)

Ages 60–69			Ages 70–79			Ages 80–89		
Age	Male	Female	Age	Male	Female	Age	Male	Female
⋮	⋮	⋮						
60	0.005094	0.003508	70	0.011352	0.009200	80	0.033220	0.025165
61	0.005611	0.003971	71	0.012413	0.010047	81	0.037784	0.028948
62	0.006166	0.004470	72	0.013670	0.010977	82	0.042847	0.033195
63	0.006756	0.005002	73	0.015144	0.012003	83	0.048099	0.038387
64	0.007395	0.005583	74	0.016852	0.013153	84	0.054355	0.044356
65	0.008103	0.006231	75	0.018806	0.014480	85	0.061550	0.050743
66	0.008544	0.006642	76	0.021021	0.016018	86	0.069919	0.058172
67	0.009073	0.007136	77	0.023529	0.017793	87	0.079621	0.066319
68	0.009704	0.007733	78	0.026364	0.019854	88	0.090101	0.075174
69	0.010458	0.008426	79	0.029559	0.022275	89	0.102551	0.084097
						⋮	⋮	⋮

Using the selected mortality table, a tontine administrator could publish its 2019 nominal tontine yields (illustrated in **Table 2**) for each age and gender cohort. These values are determined very simply by looking up the corresponding mortality rate, q, for each cohort from Table 1 and applying the formula $r = q/(1 - q)$. **Figure 1** plots the nominal yields for 2019 for a wider range of ages on a logarithmic scale to show that they increase exponentially with age. Because mortality rates are relatively low at younger ages, nominal tontine yields are likewise relatively low at younger ages. As a result, the benefit of investing in a tontine is marginal during most of the typical working years, becoming more significant as the typical retirement age approaches and eventually very large at more advanced ages.[13]

It is not uncommon for employee benefit plan regulations to require that men and women be treated equally with respect to benefits and, therefore, to mandate that annuitized payouts be made using blended unisex mortality rates. In this case, blended unisex mortality rates might also be required of an employee benefit tontine pension.[14]

An interesting consideration in the era of genetic testing and "big data" is the potential for life expectancy rates to be identified and

[13]Note carefully that Table 1, Table 2, and Figure 1 are all as of 2019. Because mortality at ages up to 104 is projected to improve in future years, the mortality rates and nominal yields for each cohort below age 104 will be somewhat lower in future years to reflect the expectation of fewer deaths at each age up to 104.

[14]Mandated gender-neutral payout equality introduces an actuarially unfair bias to the benefit of women over men because women tend to live longer. This bias inherently violates the fair tontine principle discussed earlier. In this case, we might redefine the term "fair" to mean "as fair as possible given regulatory requirements."

TABLE 2. NOMINAL TONTINE YIELD TABLE FOR 2019 (EXCERPT)

Ages 60–69			Ages 70–79			Ages 80–89		
Age	Male	Female	Age	Male	Female	Age	Male	Female
⋮	⋮	⋮						
60	0.5120%	0.3520%	70	1.1482%	0.9285%	80	3.4361%	2.5815%
61	0.5643	0.3987	71	1.2569	1.0149	81	3.9268	2.9811
62	0.6204	0.4490	72	1.3859	1.1099	82	4.4765	3.4335
63	0.6802	0.5027	73	1.5377	1.2149	83	5.0529	3.9919
64	0.7450	0.5614	74	1.7141	1.3328	84	5.7479	4.6415
65	0.8169	0.6270	75	1.9166	1.4693	85	6.5587	5.3455
66	0.8618	0.6686	76	2.1472	1.6279	86	7.5175	6.1765
67	0.9156	0.7187	77	2.4096	1.8115	87	8.6509	7.1030
68	0.9799	0.7793	78	2.7078	2.0256	88	9.9023	8.1284
69	1.0569	0.8498	79	3.0459	2.2782	89	11.4269	9.1819
						⋮	⋮	⋮

underwritten more carefully than simply by age, gender, and country of residence. Perhaps one day such simple mortality tables will be rendered obsolete and mortality rates will be assessed using other factors as well. This change would presumably reduce the degree of adverse selection that exists for both annuities and tontines.

FORFEITURE ALLOCATION

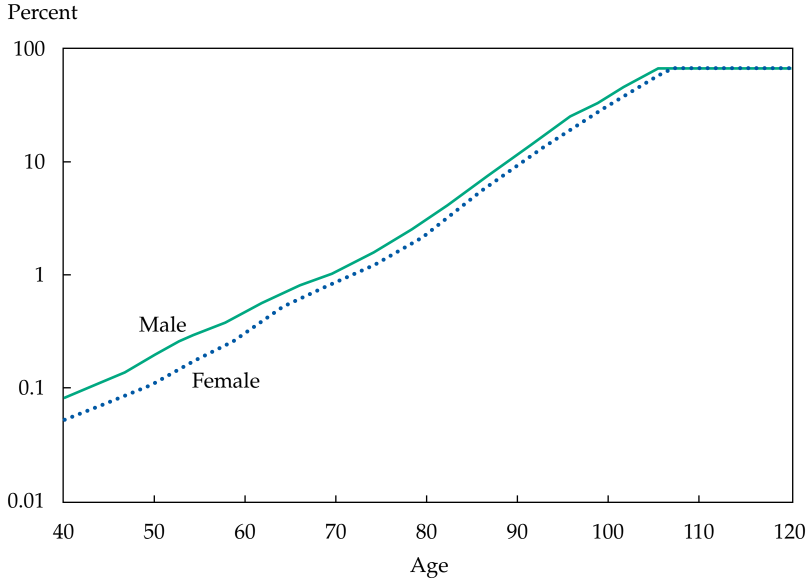
Surviving members of a fair tontine do not receive equal allocations of each dying member's forfeited balance. Rather, they receive unequal allocations that depend on their respective mortality rate, q, and account balance, s. These portions must be carefully calibrated to ensure that the fairness constraint holds for all members. Various methods have been proposed; for the purpose of this text, I elect to illustrate using the "nominal-gain method" described in Sabin and Forman (2016), which is desirable for its relative simplicity and explanatory properties.[15]

One useful property of this method is that it promotes transparency by being easily decomposed into two simple components: (1) a nominal tontine yield for each member, which is easily obtained from the tontine's mortality table, and (2) a common adjustment factor that accounts for the difference between the amount of forfeitures actually experienced by the pool during a given period and the amount that was nominally expected.

The first component was discussed previously. A member's nominal tontine yield is $r = q/(1 - q)$,

[15]The authors explained that the nominal-gain method is not precisely fair but showed that the imprecision is negligible, so for practical purposes, the method can be regarded as fair. Precisely fair methods can be found in Sabin (2010) and Donnelly et al. (2014), but they are much more complicated. For additional methods, see the references given in the Literature section.

FIGURE 1. ANNUAL NOMINAL TONTINE YIELDS FOR 2019 (LOGARITHMIC SCALE)



where q is the member's probability of dying during the period, which depends on the member's age and gender. The value r is very transparent; it would be published in advance by the tontine administrator as illustrated in the example of Table 2.

The second component—the common adjustment factor, known as the *group gain*, G—is found by dividing the sum of the balances of those who died during the period by the sum of the nominal tontine gains of those who survived the period. If we let D denote the subset of members who died during the period (the decedents), the group gain is

$$G = \frac{\sum_{j \in D} s_j}{\sum_{j \notin D} r_j s_j}.$$

In the parlance of fair games, the group gain represents the ratio of the actual losses to the expected gains. By definition of the fair tontine principle, the expected value of the group gain, G, in a fair tontine is 1.[16] However, the actual group gain will randomly deviate from 1 from period to period as a function of who actually dies. If the total amount forfeited by decedents is greater than the total nominal gains expected by survivors, then G will be greater than 1 and the survivors will all receive more than the nominal gains they anticipated. Conversely, if the total amount forfeited by the decedents is less than the total of the nominal gains expected by survivors, G will be less than 1 and the survivors will receive less than the nominal gains they anticipated.

Another useful property of this method is that it is readily perceived as equitably fair. Every member can anticipate receiving a tontine gain that

[16]Because the nominal-gain method is not precisely fair (see footnote 15), the expected value of the group gain is not precisely 1, but for practical purposes, it can be taken to be 1.

is close to their own nominal amount, *rs*, and can rest assured that all members are treated the same with respect to the group-gain adjustment. For example, if a member's actual tontine gain for a period turns out to be 2% less than anticipated (G = 0.98), she can rest assured that the actual tontine gain of all other surviving members will likewise be 2% less than they had anticipated.

RISK POOL OWNERSHIP CONSTRAINT

In addition to the fairness constraint discussed earlier, tontines must abide by a "risk pool ownership" constraint. For the principle of fairness to hold, no member can hold so much of the "risk pool" (a function of probabilities of death and account balances) that he would be under-compensated even if he were to receive 100% of any forfeitures. A general rule of thumb may be that no member may hold more than half of the risk pool at any time.[17] This amount cannot be known with certainty in advance because neither member deaths nor future account balances are knowable.

One way to mitigate this issue is to place a conservative limit on the maximum contribution amount such that it is unlikely any member would ever exceed the risk pool ownership constraint in the future. In addition, a provision could be established to make an additional payout distribution to any member who ends up owning more than half of the risk pool (which, while perhaps unlikely, could happen in the event that other members die faster than expected or a long-lived member achieves superior investment returns).

The risk pool ownership constraint is less likely to be an issue in open-end tontines because contributions by new members push down the portion of pool risk that is held by existing members.

TONTINES COMPARED WITH TRADITIONAL INVESTMENT PORTFOLIOS

The total return of a tontine investment is a function of two components: (1) the investment returns and (2) the amounts credited to survivors when other members forfeit their accounts at death (i.e., the tontine gains).

As discussed previously, each member can anticipate that the probabilistically expected total amount of tontine gains received will equal the probabilistically expected amount that will be forfeited if she dies.[18] Thus, the expected *net* total tontine gain is zero, meaning that the expected return on any given investment will be the same whether investing in a tontine or a regular account.

Recall now the observation made previously that the expected value of playing a fair game is the same as the expected value of not playing it. If the expected return from investing in a tontine is the same as the expected return from investing in a regular account, why invest in a tontine? The answer is that investing in a tontine changes the *conditional distribution of outcomes* in a very useful way: Those who live long lives do better by participating in the tontine,

[17]Sabin (2010) provided a complete discussion with respect to the precisely fair methodology used therein. The precise application of this constraint with respect to the nominal-gain method (or indeed any method) is complicated and beyond the scope of this text. As an aside, this issue also applies to insurance companies, which must ensure that their risk exposure is never overly concentrated in any single policyholder.

[18]Or "*when* she dies" in the case that payouts are to run for the rest of the member's life.

whereas those who die early do worse. For those who are relying on a portfolio to generate income over a highly uncertain and potentially long remaining lifetime, this outcome is exactly what they should want.[19]

TONTINES COMPARED WITH INCOME ANNUITIES

Unlike annuities, tontines guarantee nothing. Fixed-income annuities guarantee some assumed interest rate, whereas variable-income annuities make no such guarantee but, rather, guarantee payouts according to some benchmark or formula. Both types of annuity provide guarantees that cover both the idiosyncratic and systematic components of longevity risk. So, tontines are most like variable-income annuities, except that tontines alleviate only the idiosyncratic component of longevity risk. Tontine members collectively bear systematic mortality risk.

Because annuity providers (i.e., insurers) bear systematic longevity risk and back it with a guarantee, they are required to ensure their solvency by pricing in a suitable risk premium. Since tontines offer no such risk transfer or guarantee, no such risk premiums are charged. As a result, annuity purchasers sacrifice some amount of yield as the price for transferring the systematic component of longevity risk to the insurer. Tontine members keep this yield for themselves.

One can readily approximate the nominal yield advantage of a tontine to a comparable variable-income annuity by examining the annuity's reserve loading. For example, actuaries have established the Individual Annuity Reserving (IAR) mortality table by taking the expected mortality rates given in the IAM table (shown in Table 1) and reducing them to approximately 90% of the corresponding IAM rates. Thus, the annuity reserve reduces the nominal mortality credit yield, r, of an annuity relative to that of a tontine by approximately $\frac{q}{(1-q)} - \frac{0.9q}{(1-0.9q)}$, or a little over 10% each year. To illustrate, the nominal yield for 2019 of a 70-year-old male as shown in Table 2 would drop from 1.1482% to 1.0322%, whereas that of an 85-year-old male would drop from 6.5587% to 5.8644%. Note that the tontine's nominal mortality credit yield advantage can become significant when compounded over many years.[20]

TONTINE PAYOUTS

Tontines may use a wide variety of payout methods. Examples include lifetime payouts similar to immediate annuities, deferred lifetime payouts similar to longevity insurance, or payouts over a specified period similar to term annuities. Using standard actuarial techniques, payouts could be designed such that the anticipated payout amounts are either flat or growing at some specified rate, whether positive/increasing or negative/decreasing. Of course, actual payouts will be random because the investment returns and tontine gains will be random.

Annuity-Like Payouts

Annuity-like payouts could be for life or for some selected fixed term. If the account is jointly owned, payouts could continue until the last joint owner dies. Because tontines do

[19]Of course, those who have strong bequest motives or liquidity preferences may prefer regular accounts over tontines. It is unlikely that anyone would want to put all their wealth into a tontine (or an annuity for that matter).

[20]See Fullmer and Sabin (2018) for a more thorough examination of the IAR's reserve loading and the tontine's nominal yield advantage. See also Milevsky et al. (2018) for a theoretical historical analysis using historical bond yields and historical annuity yields in Canada.

not offer guarantees, promises of fixed payout amounts are not possible. Instead, payouts must be variable, similar to a variable-income annuity. Naturally, the degree of variability will depend on the volatility of the selected investment portfolio, as well as on the mortality experience of the pool. Those desiring payouts that are less volatile should select portfolios with returns that are expected to be less volatile, as well as tontine pools with a large number of members.

To illustrate how such a payout might work, consider a tontine that allows members to receive a monthly payout for life. The payout for a month is computed as s/a, where s is the member's account balance at the end of the month (after all investment income and tontine gains are applied) and a is the member's current "annuity factor." The annuity factor represents the present value of $1 paid monthly for the duration of the member's lifetime, with future payments discounted to the present using some assumed annual interest rate (AIR) that is selected when the member initially joins the tontine.[21] If it happens that the investment return exactly matches the assumed interest rate every month and the tontine's group gain is exactly 1 every month (meaning that the total amount forfeited by deceased members exactly equals the amount that was expected to be forfeited each month), then the member's payout will have the same value of s/a each month for the rest of the member's life.[22] Of course, investment returns will not exactly match the assumed interest rate, and group gains will not be exactly 1. Thus, future payouts will vary according to actual investment returns and actual mortality experience.

Members who desire their payouts to escalate (for example, to help offset inflation) could set the AIR lower than the expected return on the investment portfolio. Doing so would have the effect of increasing the annuity factor, thus lowering the initial payout but also leading to an expectation of growing payouts in the future.

It is worth noting that Milevsky and Salisbury (2015) presented a framework for shaping tontine annuity payouts that incorporates parameters to account for individual longevity-risk aversion and subjective health status. Although their framework assumes a closed-end pool with homogeneous membership (same age, gender, and investment amount), the results and illustrations are nevertheless insightful.

Other Types of Payouts

Of course, tontine payouts do not have to be annuity-like. The payout may be as simple as a single lump sum paid to the member if he is alive on some specified date in the future. In fact, most any payout method is possible as long as it is agreed to at the start, when the member first joins the tontine. In addition, it must be understood that the exact amount of the payouts is not guaranteed.

Tontine designers may be tempted to "smooth" payouts to reduce their volatility. An example is to prop up payouts when investment performance is poor on the basis of an expectation that better investment performance in the future will

[21]The formula for the annuity factor at age x is $a = \ddot{a}_x = 1 + \sum_{t=1}^{\infty} v^t \, {}_tp_x$, where ${}_tp_x$ is the probability of surviving to age $x + t$ given that the member is alive at age x and $v = 1/(1 + i)$ is the discount factor, with i being the assumed interest rate. The value of ${}_tp_x$ is calculated from the mortality table.

[22]This condition is true only up to the maximum age given by the mortality table, which in the case of the IAM table is age 120. If any member begins to approach that age, the tontine administrator would likely adjust the table to accommodate the possibility of surpassing it. Alternatively, a tontine provider might establish a policy of terminating the membership of anyone who attains the maximum age and returning any remaining account balance when that occurs.

make up for it. Although there is no technical reason that this cannot be done, it must be understood that the smoothing strategy adds risk. If the hoped-for future performance does not materialize, then payouts will eventually have to fall.[23] In that case, smoothing would have traded a slow gradual fall for a sudden hard fall. A tontine that artificially props up its payouts in this way would essentially be economically underfunded relative to its future payout objectives. A tontine designed to be fully funded at all times can allow no such smoothing—that is, unless it sets aside an artificial "smoothing reserve" that can be dipped into as needed to prop up payouts that would otherwise have fallen. In essence, such a reserve makes the tontine economically overfunded. It would not become economically underfunded as long as payout smoothing ceases once the reserve runs dry.

STRUCTURED PAYOUTS

If the goal is to minimize payout volatility, it may be desirable to construct the tontine portfolio to meet a goal of structured payouts. The idea here is one of cash flow matching. Of course, actual payouts will vary depending on actual mortality experience, but if the portfolio is structured properly, the *nominal* payout will be unaffected by portfolio returns, changes in interest rates, and reinvested tontine gains. Here, the term "nominal payout" refers to the expected payout when the group gain, G, is 1 (i.e., when actual mortality experience matches expected mortality experience).

It may seem surprising that this is possible, given that tontines make periodic tontine gain distributions (i.e., mortality credits) that get reinvested into surviving members' accounts. Would not these reinvested distributions result in reinvestment risk, defeating the ability to create structured payouts? The answer is no.

To understand why, note that tontine gains are proportional to account market values because a member's tontine gain (both the nominal-gain and group-gain components) is computed as a linear function of a member's current account balance. As a result, tontine gains will always purchase the same number of bond shares when reinvested regardless of whether market values are low or high. A simple bond ladder can be used to illustrate this.

Traditional Bond Ladder

A bond ladder is a portfolio of bonds structured to produce a precise set of cash flows when held to maturity. Treasury bonds are useful for this purpose because they minimize default risk, making the cash flows as secure as possible. Although the market value of the bonds held will vary as interest rates change over time, their cash flows will be unaffected.

Consider the case of a T-year bond ladder. For simplicity, let us assume these bonds are free of default risk and available in zero-coupon form, with maturities from one to T years, and that transactions are frictionless.[24] Suppose that the goal is to purchase T of these bonds in such a way that the cash flow received is the same every year.

Doing so is straightforward outside of a tontine. For each dollar of cash flow desired at the end of each year t, we purchase a zero-coupon bond that matures at time t with a par value of $1. The redemption value of the bond at the end of the year t will thus be $1. The price of the bond

[23] See Waring and Siegel (2018) for a broad discussion of the risks, including the risk of ruin, introduced by payout smoothing.

[24] Frictionless means the "bid" and "ask" market prices for a bond are the same.

when purchased is $1/(1 + i_t)^t$ dollars, where i_t is the current t-year spot rate. If the ladder is constructed using conventional Treasury bonds, the cash flows will be defined in absolute amounts. If the ladder is constructed using Treasury Inflation-Protected Securities (TIPS), the cash flows will be defined in real amounts—that is, adjusted for inflation.

Tontine Bond Ladder

Fullmer and Sabin (2019) showed that the required par value of the bonds can be reduced if this strategy is followed inside a tontine because the investor will enjoy a tontine yield in addition to a bond yield. The par amount one needs to purchase for the bond that matures at the end of year t is ${}_tp_x$, which represents the probability that a purchaser of age x survives to the end of year t.[25] In other words, the par amount of each bond is discounted by the probability of surviving long enough to collect that bond. The purchase price of the bond is ${}_tp_x/(1 + i_t)^t$ dollars.

This buy-and-hold bond ladder strategy works as advertised in a regular investment account since it has no default risk and no reinvestment risk because cash flows are never reinvested. Inside a tontine, however, cash flows from tontine gains are reinvested each time they are received. Yet, it turns out that this reinvestment has no effect on the payouts.

To see why, think of each bond as a separate investment that pays out a single lump sum at maturity. In a tontine, the payout on a bond maturing at the end of year t (i.e., the sum of the bond's redemption value plus the tontine gain on it) is

$$d_t \times (1 + r_1 G_1) \times (1 + r_2 G_2) \times \cdots \times (1 + r_t G_t),$$

where d_t is the bond's original par value, $r_t = q_t/(1 - q_t)$ is the nominal tontine gain for year t, and G_t is the group gain for year t. When the tontine pool is large, the variance of G_t is low and thus G_t is likely to be close to its mean value of 1. When $G_t = 1$, the at-maturity tontine payout simplifies to

$$d_t \times (1 + r_1) \times (1 + r_2) \times \cdots \times (1 + r_t).$$

Say that the par value of a bond is $1,000. Suppose that a member's actual tontine yield for a period is $r_t G_t = 10\%$. If the current market value of the bond, s_t, equals the par value, the tontine gain will be $s_t r_t G_t = \$100$. This gain is reinvested in more of the same bond, leaving the investor with bonds worth a total par value of $1,100. But if the current market value, s_t, is only $900, for example, then the tontine gain is $90. This gain is reinvested in more of the same bond for a total current market value of $990, which likewise equates to a final par value at maturity of 990/900 × $1,000 = $1,100. Similarly, if the market value is, say, $1,100, then the tontine gain is $110, which is reinvested in more of the same bond for a total current market value of $1,210, which again equates to a final par value at maturity of 1,210/1,100 × $1,000 = $1,100. Regardless of the market price at the time of reinvestment, the new total par value always equals the original par value plus the tontine gain.

So, the amount of each bond one is required to purchase inside a tontine is the same as the amount required outside a tontine, discounted by the probability of surviving long enough to collect that bond. This amount turns out to exactly offset the nominal tontine gains.

This principle applies to each bond that is purchased in the ladder. Consider the simple case of the one-year bond. The probability of living

[25]The formula for ${}_tp_x$ is $\prod_1^t (1 - q_t)$.

to receive the payout is $1 - q_1$, so we purchase a one-year bond with par value equal to $1 - q_1$ for every dollar of cash flow desired at maturity. At maturity, we receive the par value of $d_1 = 1 - q_1$ from the bond and $q_1/(1 - q_1)d_1$ as a tontine gain distribution. Thus, the total payout is

$$\begin{aligned}
&= \overbrace{d_1}^{\text{Par value}} + \overbrace{d_1 \frac{q_1}{1-q_1}}^{\text{Tontine gain}} \\
&= (1-q_1) + (1-q_1)\frac{q_1}{1-q_1} \\
&= (1-q_1) + q_1 \\
&= 1
\end{aligned}$$

plus or minus any drift due to mortality experience when $G \neq 1$.

Now consider the case of the two-year bond. The probability of living to receive the payout is $(1 - q_1)(1 - q_2)$, so we purchase a two-year bond with a par value of $d_2 = (1 - q_1)(1 - q_2)$ for every dollar of cash flow desired at maturity. At the end of Year 1, we receive a tontine gain distribution equal to $q_1/(1 - q_1)d_2$ and reinvest it. At maturity, we receive the par value of the original bond purchase, the par value of the reinvested tontine gain from Year 1, and the Year 2 tontine gain distribution. Thus, the total payout is

$$\begin{aligned}
&= \overbrace{d_2}^{\substack{\text{Original} \\ \text{par value}}} + \overbrace{d_2\left(\frac{q_1}{1-q_1}\right)}^{\substack{\text{Par value of} \\ \text{Year 1 tontine gain}}} + \overbrace{\left[d_2 + d_2\left(\frac{q_1}{1-q_1}\right)\right]\left(\frac{q_2}{1-q_2}\right)}^{\substack{\text{Year 2} \\ \text{tontine gain}}} \\
&= d_2\left(1+\frac{q_1}{1-q_1}\right)\left(1+\frac{q_2}{1-q_2}\right) \\
&= d_2\left(\frac{1}{1-q_1}\right)\left(\frac{1}{1-q_2}\right) \\
&= (1-q_1)(1-q_2)\left(\frac{1}{1-q_1}\right)\left(\frac{1}{1-q_2}\right) \\
&= 1
\end{aligned}$$

plus or minus any drift due to mortality experience when $G \neq 1$.

Generally, we purchase $d_t = (1 - q_1)(1 - q_2) \ldots (1 - q_t)$ of the t-year bond, for a payout of

$$d_t\left(\frac{1}{1-q_1}\right)\left(\frac{1}{1-q_2}\right)\ldots\left(\frac{1}{1-q_t}\right) = 1.$$

So, virtually risk-free bond ladders can be created both in regular accounts and in tontine accounts. In a tontine, however, the payouts will vary somewhat because of the uncertainty of the tontine gains (specifically, the group gain). Recall, however, that tontine gains are never negative; people die, but they never "un-die." Thus, the tontine has the advantage of paying a tontine yield on top of the bond investment yield. The tontine effectively yields a higher return on investment with no additional investment risk. This higher yield is akin to a free lunch for the investor that is conditionally "paid for" by the investor's heirs or beneficiaries, who lose the right to any inheritance should the investor die before a bond matures.

TONTINE ACCOUNTING ILLUSTRATED

The accounting for a tontine can be presented transparently in a straightforward manner that accounts for all cash flows affecting a member's account. Consider a simplified example of a tontine pool in which tontine gains and payouts are computed and distributed yearly. Payouts are in the form of a life annuity with an assumed interest rate of 4%. Forfeiture redistributions are computed on the basis of member survival status at the end of each calendar year.[26]

[26]In practice, forfeitures would likely be processed with a lag to allow the tontine administrator time to discover who died during the previous calendar year. For a more detailed

FIGURE 2. SAMPLE ACCOUNT STATEMENT

Tontine Statement As Of: December 31, 2019

Account Overview

Value on December 31, 2018	$ 115,615.54
Market appreciation/depreciation	$ 1,174.06
Dividends, interest, and capital gains	$ 1,402.28
Balance before tontine gain	$ 118,191.88
Tontine gain	$ 1,625.73
Balance before payout	$ 119,817.61
Tontine payout	$ (9,991.95)
Value on December 31, 2019	$ 109,825.66

Tontine Gain (if alive as of December 31, 2019)

Your Nominal Tontine Yield for 2019	0.013859
× Common Group Gain Adjustment for 2019	0.992471
= Your Actual Tontine Yield	0.013755
× Your Balance Before Tontine Gain	$ 118,191.88
= Your Tontine Gain	$ 1,625.73

Payout Option: Single Life Annuity 8.3393%
(based on male born July 1, 1947)

The Common Group Gain Adjustment for 2019 was computed as follows:

Balance forfeited by those who died in 2019	$ 32,520,980.33
÷ Anticipated nominal gains of those who survived 2019	$ 32,767,688.25
= Group Gain Adjustment for 2019	0.992471

The Group Gain Adjustment for 2019 is less than one, meaning that the membership survival rate for the year was a little higher than expected.

Consider a 72-year-old male who established a tontine account on or before 1 January 2019 and survives to the end of calendar year 2019. By surviving, he is eligible to receive his share of the pool's yearly tontine gains along with his stipulated payout. His account statement as of 31 December 2019 appears in **Figure 2**.

The statement shows a set of summarized transaction entries reflecting a beginning-of-period balance from the prior year, market appreciation or depreciation, credited investment income, and realized capital gains over the year. Added together, these give an intermediate end-of-period balance of $118,191.88, before the effect of any tontine processing. Up to this point, the statement reflects only investment activity and looks like a regular investment account statement.

The shaded boxes in Figure 2 show how his tontine gain is computed. First, his nominal tontine yield of 0.013859 is selected by looking up the appropriate value for his cohort from Table 2, shown previously. This value is multiplied by the common group-gain adjustment factor for 2019 of 0.992471 to find his actual tontine yield of 0.013755. The group-gain adjustment was calculated by dividing the total balance forfeited by those who died during the year by the total nominal gains of those who survived the year, as shown in the shaded box at the bottom of the statement. Thus, the member is credited with an actual tontine gain of 0.013755 × $118,191.88 = $1,625.73. Adding this amount gives an updated balance of $119,817.61.

Next, his payout is calculated on the basis of his selected payout option of a single life annuity as discussed previously in the Tontine Payouts section. In this case, the 2019 life annuity payout rate for a 72-year-old male is 8.3393%, so his payout is $119,817.61 × 0.083393 = $9,991.95. Subtracting this amount gives an end-of-period account balance of $109,825.66.

This is just one example of how membership in a tontine might be accounted for. One may also imagine a unitized method of accounting that could omit some of the detail on member statements. This might make the statement even simpler, albeit somewhat less transparent.

example that considers such a lag, refer to Fullmer and Sabin (2018).

TONTINE STRUCTURES

A number of different tontine structures or schemes are possible.

Tontine Funds

When people think of tontines, they typically think of a collective investment pool with a common portfolio and a common payout method—in other words, a type of managed payout mutual fund that offers mortality credits. In this way, the portfolio strategy might be optimized with respect to some given payout objective.

The pool could be either closed-end or open-end. It could accept either a homogeneous or heterogeneous membership. An example of a homogeneous membership pool is one that accepts only members of a specific gender and age. Heterogeneous membership pools might accept members of any gender and a wide range of ages. Assuming that any design adheres to the fair tontine principle, they would all be equally fair.

The fund would specify its payout algorithm, whether in the form of a life annuity, a deferred annuity, a term annuity, a lump sum on some given date, or something else. The portfolio could be conservative or aggressive, with a direct and obvious impact on payout volatility.

Tontine Pensions

The slow decline of defined benefit pensions has been fueled in large part by a recognition of the risks and high cost of providing fixed guarantees of lifetime income. Forman and Sabin (2015) explored the concept of a tontine pension alternative that would reduce plan sponsor risks and costs by eliminating the guarantee. Indeed, a tontine pension would always be fully funded because payouts would vary depending on investment performance and mortality experience. Presumably, the investment strategy would seek to minimize payout variability. It is possible that longevity bonds could also be held in the portfolio to help mitigate systematic longevity risk should a sponsor so desire.[27]

Tontine Brokerage Accounts

The fair tontine principle does not limit a tontine's design to a common investment portfolio or payout method. It is also possible to design fair tontine *brokerage* accounts in which anyone may join at any time, contribute as they wish, invest as they wish, trade as they wish, and elect payouts as they wish within some broad menu of options offered by the administrator. In this way, members could customize their experience to their unique needs and preferences. This idea is explored in Fullmer and Sabin (2018).

ADDITIONAL TOPICS

Tontine developers should also consider a number of other factors when designing these products.

Allowance for Bequests

Ordinarily, a tontine investment is wholly forfeited at death. Those with a bequest motive might allocate a portion of their retirement saving to a tontine account and the rest to a regular account. The strength of the bequest motive (among other things) then dictates how much goes into each type of account.

It is possible, however, to design a tontine that pays some percentage of a member's account

[27]Longevity bonds and other forms of longevity derivatives make payouts that are based on the survivorship of a given population group.

balance at death to the member's beneficiaries, with the remainder redistributed to the surviving members of the tontine pool.[28] Naturally, this would reduce the tontine yields enjoyed by survivors. Still, some consumers might find this trade-off appealing.

Pool Breadth

Another design consideration has to do with the "breadth" of the pool. One school of thought is that the pool should be as large as possible to maximize the diversification of idiosyncratic longevity risk and thereby minimize the difference between actual tontine gains and expected tontine gains. To an economist, this certainly makes sense. A pool might be nationwide or perhaps even span multiple countries.

Not all consumers might think about it this way. Some might balk at broad pools for social reasons. For example, a villager might prefer a smaller tontine pool that confines its membership to her village. Although a small localized pool would offer less diversification, she might prefer the comfort of knowing that her death will benefit her fellow villagers whom she knows and cares about, rather than complete strangers.

Proof of Life

A tontine administrator must maintain an effective method of identifying which members have died and which have survived. Biometric technologies are often suggested as solutions. For example, payouts might be withheld until members log in to a computer or mobile phone application and positively identify themselves as being alive (e.g., using facial recognition technology). Because such biometric technologies are imperfect, however, it seems prudent that they be backed up by periodic queries to official government death records.

A procedure should also be in place to address situations in which it is discovered that members had been misidentified as alive or deceased. If a deceased member is misidentified as alive, he will have improperly received payouts that should have been allocated to the true survivors. If the tontine administrator can claw these improper payments back, the monies may then be reallocated to the survivors properly. If a living member is misidentified as deceased, her account should be restored by reallocating back to her the improper forfeiture from her account that went to others.

Taxation

Tax policymakers will need to determine how tontine gains, forfeitures, and payouts should be treated for tax purposes. One possibility is to follow the established rules for annuities, but that is not the only possibility. Tontines and annuities are fundamentally different, and it is not inconceivable to think that they would be taxed differently.

Financial Planning and Wealth Management

Should tontines make a comeback, they will represent an entirely new class of investment vehicle that lies somewhere between regular investments and annuities, bearing elements of each. This innovation has implications for the financial planning community in terms of wealth allocation between product types. Models that have been developed for allocating wealth between regular investments and annuities would presumably be updated to include tontines as an additional product type choice.

[28]For example, refer to Bernhardt and Donnelly (2019).

Perception and Objections

The word "tontine" suffers a negative connotation in the minds of many. Some have even suggested that the products create a nefarious moral hazard by incenting members to kill other members. Tontines have likewise been the subject of a good deal of graveyard humor in popular literature and film, including *The Wrong Box* (an 1889 comic novel by Robert Louis Stevenson and Lloyd Osbourne, turned into a film in 1966). Fiction aside, this kind of moral hazard can be mitigated by designing the pool with the goal of achieving wide breadth and depth of ownership (e.g., heterogeneous membership, perpetually open to new members), using annuity-like payouts as opposed to a "winner-take-all" lottery payout, keeping member identities private, and using autonomous technologies (e.g., blockchain) to handle forfeiture and payout transactions.

Some may object to the fact that a dying member's money does not benefit his heirs. This issue can be partially addressed by allocating some portion of a dying member's account balance to that member's chosen beneficiaries. The effect would be to lower the tontine yields of survivors.

Another objection concerns the fact that tontines are generally irrevocable and, therefore, are an illiquid asset. This characteristic is not new; payout annuities are generally irrevocable, too. There may be ways around this illiquidity issue, such as those Weinert (2017) contemplated, although the trade-off might be steep.

On the one hand, it is easy to say that tontines should be kept "pure," rather than accommodating bequests or liquidity needs, and to argue that monies for bequest goals and liquidity needs should be held separately in a regular account. On the other hand, individuals may be ill equipped to make such allocation decisions on their own and might not have enough wealth to afford a professional investment adviser. Perhaps "watered-down" tontines have a place in the world, too.

Given that tontines are widely misperceived and have been given a bad reputation by their past, some suggest giving up on the name "tontine" and replacing it with some new name, such as *survivor pool* or even *Hamiltonian* (in honor of Alexander Hamilton, who once proposed a form of tontine to finance US government operations).[29] In honor of the man who preceded Tonti with the idea in the first place, I will throw my hat in the ring, with tongue in cheek, by suggesting that the new modern tontine be called the *bourey*.

CONCLUSION

Tontines are a more than 350-year-old invention that were once very popular before being virtually regulated out of existence owing to the behavior of a few bad actors who (mis)managed them over 100 years ago. They never completely went away, however, and now appear to be on the verge of making a comeback. Tontines offer a unique value proposition and a new choice. Choice is good for society. Certainly, the global retirement challenge is in need of solutions.

Tontines and income annuities are different solutions to the same problem. Whether the guarantee offered by an income annuity is worth the extra cost is a matter of personal preference. Although some level of variability will always exist in a tontine's payout stream, fluctuations can be made modest if the tontine invests conservatively—and perhaps uses structured cash flow matching—and maintains a sufficiently

[29]The reference to Hamiltonian is given in Milevsky and Salisbury (2015).

large number of members. Most retirees can probably accept some level of variability in their retirement income. By collectively sharing longevity risk among themselves and dispensing with the cost of guarantees, retirees may be able to achieve significant cost savings using tontines rather than annuities.

Compared with regular investment accounts, tontines provide an additional tontine gain on top of any investment return. Investment returns can be either positive or negative, but tontine gains are never negative. They can only improve an investor's performance. Moreover, because tontines pool longevity risk, they can offer the assurance of annuity-like lifetime payouts in a way not possible with regular investments.

The fair tontine principle is remarkably versatile. Many designs are possible, including those that allow investors much flexibility for customizing to their unique preferences and needs.

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